

# Higher spin gravity in 3D: black holes, global charges and thermodynamics

Alfredo Pérez<sup>1</sup>, David Tempo<sup>1</sup>, Ricardo Troncoso<sup>1,2\*</sup>

<sup>1</sup>*Centro de Estudios Científicos (CECs), Casilla 1469, Valdivia, Chile and*

<sup>2</sup>*Universidad Andrés Bello, Av. República 440, Santiago, Chile.*

## Abstract

Global charges and thermodynamic properties of three-dimensional higher spin black holes that have been recently found in the literature are revisited. Since these solutions possess a relaxed asymptotically AdS behavior, following the canonical approach, it is shown that the global charges, and in particular the mass, acquire explicit nontrivial contributions given by nonlinear terms in the deviations with respect to the reference background. It is also found that there are cases for which the first law of thermodynamics is fulfilled in the canonical ensemble, i.e., without work terms associated to the presence of higher spin fields, and remarkably, the semiclassical higher spin black hole entropy is exactly reproduced from Cardy formula.

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\*Electronic address: aperez, tempo, troncoso@cecs.cl

## I. INTRODUCTION

The exact higher spin black hole solutions in three spacetime dimensions that have been recently found in [1–3] provide a unique arena in order to acquire a deeper understanding of higher spin gravity [4–7]. These solutions possess a relaxed asymptotically AdS behaviour as compared with the one proposed in refs. [8–10]. It is worth pointing out that a similar effect occurs in the case of hairy black holes with scalar fields with slow fall-off at infinity [11–15]. In this case, it is known that the asymptotic conditions turn out to be relaxed with respect to the ones of Brown and Henneaux [16], and as a consequence, the global charges, and in particular the mass, acquire nontrivial contributions given by nonlinear terms in the deviation of the fields with respect to the reference background. Therefore, it is natural to wonder about the persistence of this effect for black holes endowed with higher spin fields. Here it is shown that this is indeed the case for the class of higher spin black holes mentioned above. In fact, as they possess a relaxed asymptotic behaviour, their energy does not depend linearly on the deviation of the fields with respect to the background configuration. Therefore, the higher spin black hole mass must be computed from scratch.

In the next section, higher spin gravity in three dimensions as a Chern-Simons theory is revisited, where the canonical approach to construct conserved charges as surface integrals is also briefly discussed. Section III is devoted to perform the explicit computation of the mass, including the analysis of nontrivial integrability conditions for the higher spin black hole solution found in refs. [1, 2]. This is also carried out for the solution of ref. [3] in section IV, where it is also shown that the first law of thermodynamics is fulfilled in the canonical ensemble, i.e., without work terms associated to the presence of higher spin fields. The semiclassical entropy of this higher spin black hole is also shown to be exactly reproduced by means of Cardy formula. Final comments are discussed in section V.

## II. HIGHER SPIN GRAVITY AS A CHERN-SIMONS THEORY IN 3D

As it was shown in [17, 18], a Chern-Simons action whose gauge group is given by  $SL(N, \mathbb{R}) \times SL(N, \mathbb{R})$  describes a theory of gravity with negative cosmological constant, coupled to interacting fields of higher spin  $s = 3, 4, \dots, N$ , in three dimensions. Analogously, a consistent theory of gravity that includes the whole infinite tower of higher spin fields can

be constructed by means of two copies of the  $hs(1, 1)$  algebra. Let us focus on the simplest case that corresponds to  $N = 3$ . The theory can then be described in terms of two independent connection one-forms,  $A^+$  and  $A^-$ , associated to each copy of  $SL(3, \mathbb{R})$ , so that the action is given by

$$I = I_{CS} [A^+] - I_{CS} [A^-] \ , \quad (1)$$

where

$$I_{CS} [A] = \frac{k}{4\pi} \int_M \left\langle AdA + \frac{2}{3} A^3 \right\rangle \ , \quad (2)$$

and the level is determined by the Newton constant and the AdS radius according to  $k = \frac{l}{4G}$ . Here the bracket stands for an invariant nondegenerate bilinear form of  $SL(3, \mathbb{R})$  that is proportional to the Cartan-Killing metric. The fundamental representation of  $SL(3, \mathbb{R})$  is generated by  $L_i$  and  $W_m$ , where  $i = -1, 0, 1$ , and  $m = -2, -1, \dots, +2$ , and the bracket is given by a quarter of the trace, i.e.,  $\langle \dots \rangle = \frac{1}{4} \text{tr}(\dots)$ , see e.g., [9].

Since the field equations imply the vanishing of  $SL(3, \mathbb{R})$  curvatures, i.e.,  $F^\pm = 0$ , the connections become locally flat on shell.

It is useful to introduce a generalization of the dreibein and spin connection according to

$$A^\pm = \omega \pm \frac{e}{l} \ , \quad (3)$$

so that, in the principal embedding of  $sl(2, \mathbb{R})$  into  $sl(3, \mathbb{R})$ , the spacetime metric and the spin 3 field are recovered from

$$g_{\mu\nu} = \frac{1}{2} \text{tr}(e_\mu e_\nu) \ , \quad (4)$$

and

$$\varphi_{\mu\nu\rho} = \frac{1}{3!} \text{tr}(e_{(\mu} e_\nu e_{\rho)}) \ , \quad (5)$$

respectively.

It is worth pointing out that the metric transforms nontrivially under the higher spin gauge symmetries embedded in  $SL(3, \mathbb{R}) \times SL(3, \mathbb{R})$ , and as a consequence, some standard geometric and physical notions may appear to be ambiguous, since they are no longer gauge invariant. Therefore, in order to provide a reliable definition of energy, it is better to stay attached to its very basic definition: the energy corresponds to the conserved charge associated with the generator of time evolution, i.e., it is given by the Hamiltonian.

## A. Canonical generators

The suitable definition of energy we look for a Chern-Simons theory of the form (1) can then be obtained following the Regge-Teitelboim approach [19]. In the canonical formalism, the variation of the conserved charge associated to an asymptotic gauge symmetry generated by an algebra-valued parameter  $\eta = \eta^+ + \eta^-$  is given by

$$\delta Q(\eta) = -\frac{k}{2\pi} \int_{\partial\Sigma} (\langle \eta^+ \delta A_\phi^+ \rangle - \langle \eta^- \delta A_\phi^- \rangle) d\phi, \quad (6)$$

where  $\partial\Sigma$  stands for the boundary of the spacelike section  $\Sigma$  (see e.g., [20–22]). Since diffeomorphisms are not independent of gauge transformations for a Chern-Simons theory in three dimensions, the variation of the generator of an asymptotic symmetry spanned by an asymptotic killing vector  $\xi^\mu$ , reduces to

$$\delta Q(\xi) = \frac{k}{2\pi} \int_{\partial\Sigma} \xi^\mu (\langle A_\mu^+ \delta A_\phi^+ \rangle - \langle A_\mu^- \delta A_\phi^- \rangle) d\phi. \quad (7)$$

Therefore, the variation of the energy,  $M = Q(\partial_t)$ , is given by

$$\delta Q(\partial_t) = \frac{k}{2\pi} \int_{\partial\Sigma} (\langle A_t^+ \delta A_\phi^+ \rangle - \langle A_t^- \delta A_\phi^- \rangle) d\phi. \quad (8)$$

The variation of the canonical generators in eqs. (6), (7), and (8) then corresponds to the ones of higher spin gravity provided the parameter  $\eta^\pm$  takes values on  $hs(1,1)$  or  $sl(N, \mathbb{R})$ . In both cases, a consistent set of asymptotic conditions has been proposed in refs. [8] and [9, 10], respectively, being such that the conserved charges turn out to be linear in the deviation of the fields with respect to the  $\text{AdS}_3$  background<sup>1</sup>. Indeed, for the case of  $sl(3, \mathbb{R})$ , the asymptotic conditions for the gauge fields can be written as

$$A^\pm = \bar{A}^\pm + \Delta A^\pm,$$

where the deviation with respect to the background configuration  $\bar{A}^\pm$ , which is assumed to be  $\text{AdS}_3$  spacetime of radius  $l$ , is of the form

$$\Delta A^\pm = \pm \left( -\frac{2\pi}{k} \mathcal{L}^\pm e^{-\rho} L_{\mp 1}^\pm \mp \frac{\pi}{2k} \mathcal{W}^\pm e^{-2\rho} W_{\mp 2}^\pm \right) dx^\pm, \quad (9)$$

with  $x^\pm = \frac{t}{l} \pm \phi$ , and  $L_i^\pm$  span two copies of the  $sl(2, \mathbb{R})$  subalgebra. By virtue of (9), it is then simple to verify that the variation of the canonical generators in (7) becomes linear

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<sup>1</sup> The supersymmetric extension of the asymptotic conditions of [8] was developed in ref. [23].

in the deviation of the fields. Therefore, the conserved charges can be readily integrated so that, in particular, the zero modes of the Virasoro generators read

$$L_0^\pm = Q(\partial_\pm) = \int \mathcal{L}^\pm d\phi, \quad (10)$$

and hence, the mass (8) is given by

$$M = Q(\partial_t) = \frac{1}{l} \int (\mathcal{L}^+ + \mathcal{L}^-) d\phi. \quad (11)$$

It is also simple to verify that eqs. (10) and (11) also hold for the asymptotic conditions in [9, 10] and [8], for  $sl(N, \mathbb{R})$  and  $hs(1, 1)$ , respectively, where it has also been shown that the algebra of the canonical generators acquires the same central extension as the one found by Brown and Henneaux in [16] for General Relativity with negative cosmological constant,

$$c = \frac{3l}{2G}. \quad (12)$$

In the next section we show that, since the higher spin black hole solutions found in [1–3] do not fulfill the asymptotic conditions of ref. [9], the zero mode of their Virasoro generators, and then their mass, differ from eqs. (10) and (11), respectively, because they acquire explicit nonlinear contributions that come from the deviation of the fields with respect to the reference background. It is also worth pointing out that further integrability conditions that guarantee that the variation of the energy is an exact differential are also found. This implies that some of the integration constants appearing on the higher spin black hole solutions considered here become functionally related, and hence they are not independent.

### III. AMMON-GUTPERLE-KRAUS-PERLMUTTER SOLUTION

Let us first consider the higher spin black hole solution found in refs. [1, 2] for the case  $N = 3$ . The gauge field can be conveniently written as

$$A^\pm = g_\pm^{-1} a^\pm g_\pm + g_\pm^{-1} dg_\pm, \quad (13)$$

where  $g_\pm = g_\pm(\rho)$  stand for suitable elements of each copy of  $SL(3, \mathbb{R})$  that depend only on the radial coordinate, so that

$$\begin{aligned} a^\pm = & \pm \left( L_{\pm 1}^\pm - \frac{2\pi}{k} \mathcal{L} L_{\mp 1}^\pm \mp \frac{\pi}{2k} \mathcal{W} W_{\mp 2}^\pm \right) dx^\pm \\ & + \mu \left( W_{\pm 2}^\pm - \frac{4\pi}{k} \mathcal{L} W_0^\pm + \frac{4\pi^2}{k^2} \mathcal{L}^2 W_{\mp 2}^\pm \pm \frac{4\pi}{k} \mathcal{W} L_{\mp 1}^\pm \right) dx^\mp, \end{aligned} \quad (14)$$

correspond to the connection in the “wormhole gauge”. As explained in [1, 2], this solution does not fulfill the asymptotic conditions of ref. [9] in eq. (9), since the metric asymptotically approaches to that of  $\text{AdS}_3$ , but of radius  $\tilde{l} = l/2$ , and moreover the deviation with respect to the background configuration,  $\Delta A^\pm$ , possesses additional components along  $dx^\mp = \frac{1}{\tilde{l}}dt \mp d\phi$ .

It is simple to verify that, since  $g_\pm = g_\pm(\rho)$ , the variation of the canonical generator in eq. (8) reduces to

$$\delta Q(\partial_t) = \frac{k}{2\pi} \int_{\partial\Sigma} (\langle a_t^+ \delta a_\phi^+ \rangle - \langle a_t^- \delta a_\phi^- \rangle) d\phi , \quad (15)$$

and hence, the variation of the energy,  $\delta M = \delta Q(\partial_t)$ , is given by

$$\delta M = \frac{8\pi}{l} \left[ \delta\mathcal{L} - \frac{32\pi}{3k} \delta(\mathcal{L}^2 \mu^2) + \mu \delta\mathcal{W} + 3\mathcal{W} \delta\mu \right] . \quad (16)$$

Since the configuration is static, by virtue of (7), the variation of the zero modes of the Virasoro generators then reads

$$\delta L_0^\pm = \delta Q(\partial_\pm) = \frac{\tilde{l}}{2} \delta M . \quad (17)$$

According to eq. (16), the higher spin black hole energy not only has the expected linear contribution in  $\delta\mathcal{L}$ , but also acquires additional terms that depend nonlinearly in the integration constants. Note that the energy is well defined provided its variation becomes an exact differential, so that it can be integrated. Therefore, in order to guarantee that, the last two terms at the r.h.s. of eq. (16) give a nontrivial integrability condition that have to be fulfilled. This condition then reads

$$\delta^2 M = \frac{16\pi}{l} \delta\mathcal{W} \wedge \delta\mu = 0 , \quad (18)$$

and as a consequence, the integration constants  $\mu$  and  $\mathcal{W}$  are not independent, i.e., they become functionally related as

$$\mu = \mathcal{F}'(\mathcal{W}) , \quad (19)$$

where  $\mathcal{F}'$  is the derivative of an arbitrary function  $\mathcal{F}$  that is fixed once precise boundary conditions are provided. Hence, the mass is given by

$$M = \frac{8\pi}{l} \left[ \mathcal{L} - \frac{32\pi}{3k} \mu^2 \mathcal{L}^2 + 3\mu \mathcal{W} - 2\mathcal{F} \right] , \quad (20)$$

up to an arbitrary constant without variation.

#### IV. CASTRO-HIJANO-LEPAGE-JUTIER-MALONEY SOLUTION

The second example we consider is the static higher spin black hole solution found in [3], also for  $N = 3$ . As in the previous case, it is convenient to express the gauge field as in eq. (13), where now  $g_{\pm} = e^{\pm \rho L_0^{\pm}}$ , and

$$a^{\pm} = \pm (\ell_P L_{\pm 1}^{\pm} - \mathcal{L} L_{\mp 1}^{\pm} \pm \Phi W_0^{\pm}) dx^{\pm} + (\ell_D W_{\pm 2}^{\pm} + \mathcal{W} W_{\mp 2}^{\pm} - Q W_0^{\pm}) dx^{\mp}, \quad (21)$$

with<sup>2</sup>

$$Q \ell_P - 2 \mathcal{L} \ell_D = 0 \quad ; \quad Q \mathcal{L} - 2 \mathcal{W} \ell_P = 0. \quad (22)$$

It is simple to verify that this solution does not fit within the asymptotic conditions in eq. (9). Indeed, the asymptotic form of the metric approaches to  $\text{AdS}_3$  of radius  $\tilde{l} = l/2$ , and the deviation of the gauge field also possesses additional components along  $dx^{\mp}$ .

The variation of the zero modes of the Virasoro generators can then be obtained from eq. (17), where according to eq. (15), the mass reads

$$\delta M = \delta Q (\partial_t) = \frac{1}{3G} [\delta (3 \mathcal{L} \ell_P - 4 Q^2 - 2 Q \Phi + \Phi^2) + 4 \Phi \delta Q]. \quad (23)$$

Therefore, as expected, the variation of the mass has a nonlinear dependence in the integration constants. The integrability condition that comes from (23) is given by

$$\delta^2 M = \frac{4}{3G} \delta \Phi \wedge \delta Q = 0, \quad (24)$$

which means that the integration constants  $\Phi$  and  $Q$  are functionally related. It is convenient to express this relation as

$$\Phi = \mathcal{F}'(Q), \quad (25)$$

for some arbitrary function  $\mathcal{F}$ . Hence, up to an arbitrary constant without variation, the mass can be written as

$$M = \frac{1}{G} [\mathcal{L} \ell_P + 4 Q^2] + w(Q), \quad (26)$$

with

$$w(Q) := \frac{1}{3G} [(\Phi - 4Q)(\Phi + 4Q) - 2(Q\Phi - 2\mathcal{F})], \quad (27)$$

so that the zero modes of the Virasoro generators read

$$L_0 := L_0^{\pm} = \frac{\tilde{l}}{2} M = \frac{l}{4} M. \quad (28)$$

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<sup>2</sup> We have chosen a different orientability as compared with the one in [3], i.e.,  $x^+ \leftrightarrow x^-$ . As in the previous section, here  $x^{\pm} = \frac{t}{l} \pm \phi$ , and hereafter we will consider the branch with  $\ell_P, \ell_D \neq 0$ .

### A. Thermodynamics

As it was shown in [3], requiring the holonomy around the thermal cycle of the Euclidean solution to be trivial, allows to fix Hawking temperature according to

$$T = \frac{2}{\pi l} \sqrt{\mathcal{L} \ell_p + 4Q^2} , \quad (29)$$

and gives an additional condition that reads

$$\Phi = 4Q . \quad (30)$$

This allows to determine the precise form of the function  $\mathcal{F}$  in (25), according to  $\mathcal{F} = 2Q^2$ . As a consequence, the function  $w(Q)$  in (27) vanishes, so that the mass becomes proportional to the square of the temperature, i.e.,

$$M = \frac{\pi^2 l^2}{4G} T^2 , \quad (31)$$

and then the entropy can be readily found in the canonical ensemble,  $dM = TdS$ , to be given by

$$S = \pi l \sqrt{\frac{M}{G}} . \quad (32)$$

Note that in terms of the zero modes of the Virasoro generators, in eq. (28), the semiclassical entropy of the higher spin black hole (32) reads

$$S = 4\pi \sqrt{\frac{l}{4G} L_0} , \quad (33)$$

which exactly agrees with Cardy formula,

$$S = 4\pi \sqrt{\frac{c}{6} L_0} , \quad (34)$$

provided

$$c = \frac{3l}{2G} , \quad (35)$$

i.e., precisely the standard central charge that has been found to hold also for higher spin gravity with asymptotically  $\text{AdS}_3$  boundary conditions [8–10]. This result suggests that the higher spin black hole found in [3] could be naturally regarded as a large nonperturbative deviation with respect to the  $\text{AdS}_3$  vacuum of radius  $l$ .



## V. DISCUSSION

The canonical formalism to compute conserved charges as surface integrals [19] has been briefly reviewed in the case of higher spin gravity in three dimensions, and it was applied in order to obtain the mass of the higher spin black holes in refs. [1, 2] and [3]. In both cases, it was found that the energy acquires nonlinear terms in the deviation of the fields with respect to the reference background. This goes by hand with non trivial functional relationships between the integration constants that have to be fulfilled in order to ensure integrability of the charges. This effect occurs due to the fact that these solutions possess a relaxed asymptotically AdS behavior as compared with the ones in [8–10], and hence the zero modes of the Virasoro generators, and then their mass, differ from eqs. (10) and (11), respectively<sup>3</sup>. It is worth then pointing out that finite charges as surface integrals that are obtained through different perturbative approaches do not capture this effect. Indeed, although they may transform suitably under the Virasoro symmetry, *a priori*, there is no guarantee that they reproduce the energy unless one explicitly check that they generate the time evolution.

For the specific examples considered here, some remarks are in order. In the case of the higher spin black hole of [1, 2] the integrability condition that makes the variation of the energy to be an exact differential reduces to eq. (19). The case  $\mu = \mu_0$ , where  $\mu_0$  is arbitrary and without variation is certainly one of the possibilities, and hence along the lines of the AdS-CFT correspondence, the generic functional relationship (19) naturally arises in the context of multi-trace deformations [31, 32]. It is also worth mentioning that, as explained in [1, 2], for the Euclidean solution, requiring the holonomy around the thermal cycle to be trivial not only determines the higher spin black hole temperature, but also gives an additional condition. As a consequence, once eq. (19) is imposed, the mass in eq. (20) turns out to depend on a single integration constant. It would then be interesting to explore how thermodynamics works once these results are taken into account.

In the case of the higher spin black hole found in [3], requiring triviality of the holonomy around the thermal cycle singles out a unique possibility for the functional relationship in eq. (25). Since the mass becomes proportional to the square of the temperature, as in eq.

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<sup>3</sup> Further examples exhibiting similar features have also been discussed in refs. [24, 25] in the context of topologically massive gravity [26–28], as well as in [29] for BHT “new” massive gravity [30] in vacuum.

(31), the thermodynamics can be easily carried out in the canonical ensemble. Remarkably, the semiclassical entropy of the higher spin black hole was shown to exactly agree with Cardy formula, provided the central charge is the one of Brown and Henneaux, which was found to hold also for higher spin gravity with asymptotically  $\text{AdS}_3$  boundary conditions [8–10]. Note that if the asymptotic conditions are relaxed in a consistent way with the asymptotic symmetries, the central charge does not change, since it is determined by the ground state configuration. Therefore, although the higher spin black hole of [3] asymptotically approaches to  $\text{AdS}_3$  spacetime of radius  $\tilde{l} = \frac{l}{2}$ , our result naturally suggests that it could be consistently regarded as a large nonperturbative deviation with respect to the  $\text{AdS}_3$  vacuum of radius  $l$ . In this sense, it is worth mentioning that  $\text{AdS}_3$  with radius  $l$  has been argued to be the only ground state for which the perturbative spectrum of the theory could admit a unitary representation for large values of the central charge [33].

Curiously, in this sense, the usual practice that suggests the possibility of regarding the asymptotics of the higher spin black hole as a perturbative deviation with respect to  $\text{AdS}_3$  of radius  $\tilde{l}$  does not appear to be an appealing one. Indeed, if this was the case, according to ref. [2], the central charge that corresponds to the ground state would be given by  $\tilde{c} = \frac{3l}{8G}$ , and hence the standard form of Cardy formula in eq. (34), with  $c = \tilde{c}$ , would fail in reproducing its semiclassical entropy. It is also worth pointing out that in this case, along the lines of refs. [34, 35], Cardy formula still would have a chance to work if it is generalized so as to admit nontrivial lowest eigenvalues for the Virasoro operators, which would correspond to the ones of a suitable ground state that asymptotically approaches to  $\text{AdS}_3$  of radius  $\tilde{l}$ . However, if this was the case, the lowest eigenvalues of the Virasoro operators would be given by  $\bar{L}_0^\pm = -\frac{l}{16G} < -\frac{\tilde{c}}{24}$ , which manifestly violates the unitarity bound.

As an ending remark, it would be interesting to explore the possibility of recovering the results discussed here from the dual theory at the boundary, along the lines of [36–41], as well as inspecting whether they persist for the different class of higher spin black holes found in [42–44], or for the generic solution of [45].

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